JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 37, No. 4, July–August 2000

Statistical Analysis of Atmospheric Flight Gust Loads Analysis Data

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To establish launch vehicle loads during atmospheric flight, the statistical characteristics of the turbulence/gust-induced loads need to be determined. Recently, a Monte Carlo analysis procedure was developed that uses measured turbulence/gusts to establish launch-vehicle loads. The procedures developed to characterize the distribution of the data and to calculate tolerance bounds on these Monte Carlo loads are presented.

Nomenclature

C	= confidence level
E	= load enclosure
F_X or G_X	= cumulative distribution function of X
F_Z	= cumulative distribution function of a standard normal random variable
f_X	= probability density function of X
n	= sample size
q_p^X	= p-quantile of X
s	= sample standard deviation
T_X	= tolerance bound for X
$t_{n,\delta,p}$	= p-quantile of noncentral t distribution with n degrees of freedom and noncentrality parameter δ
X	= random variable
$\bar{x}$	= sample mean
Z	= standard normal random variable
z_p	= standard normal p-quantile

Introduction

GUST loads analyses are performed to establish launch-vehicle and space-vehicle loads as a result of turbulence encountered during atmospheric flight.[1,2] Historically, launch-vehicle loads have been calculated by applying a synthetic gust profile whose amplitude, wavelength, and shape were selected to induce loads of a desired magnitude.[3] In Ref. 3 a new Monte Carlo analysis approach that uses the turbulent component of measured wind profiles is presented. The approach requires statistical analysis of extensive loads analysis results. This paper presents the statistical analyses performed to establish a suitable distribution for gust loads so that tolerance bounds on these loads could be calculated.

The objective of this work was to characterize the statistical distribution of the gust loads data in Ref. 3 and to determine the 99.7% load enclosure. The 99.7% load enclosure is the 3-σ load enclosure for data having a normal, or Gaussian, distribution, i.e., 99.7% of normal data lie within three standard deviations of the mean of the distribution. For data with statistical distributions other than normal, however, 3-σ is not synonymous with 99.7% coverage, and the 99.7% enclosure may be significantly different from the 3-σ enclosure. Because of the uncertainty in the parameters of the statistical distribution, the 99.7% enclosure is presented as a 90% upper confidence bound on the 99.7% enclosure. In statistical terminology a 0.997/0.90 upper tolerance bound for the data is determined. Two tolerance bound procedures are described: one that assumes a gamma distribution for the gust loads and is most suitable for small sample sizes, and another that makes no distributional assumptions but is useful only if the sample size is sufficiently large.

Background

The application and theory of statistical tolerance bounds, where the derivation of tolerance bounds for normal distributions as well as for certain other distributions, are discussed.[4,5] Tolerance bounds are derived for the generalized gamma distribution, a class of statistical distributions that includes the exponential, two-parameter gamma, and Weibull distributions.[6] The procedure presented in Ref. 6, however, requires that some parameters of the distribution be known in order to calculate a tolerance bound. This procedure is not applicable to this work because all parameters of the gust-induced loads distribution need to be estimated from the data.

Characterization of Gust-Induced Loads

Gusts, which for the purposes of this discussion are defined as the nonpersistent, relatively short wavelength components of the winds[7] that a launch vehicle will encounter during flight through the atmosphere, induce loads that have been approximated by a gamma distribution.[8,9]

The distribution of a typical set of gust loads in Ref. 3 is shown in Fig. 1. The asymmetry of the data suggests that modeling the data with the normal distribution and calculating the load enclosure using 3-σ limits could significantly underpredict the true 99.7% enclosure.

Probability plots are a useful tool for assessing the distribution of data. A probability plot compares the observed data to what would be expected if the data had a particular distribution. If data have a normal distribution, the data will tend to lie on a straight line on a normal probability plot. Probability plots for other distributions can likewise be constructed, and data with a given statistical distribution will tend to lie on a straight line on the corresponding probability plot.

Normal and gamma probability plots for a typical set of gust load data in Ref. 3 are shown in Figs. 2 and 3. To create the gamma probability plot, an estimate of the shape parameter of the gamma distribution was necessary; the maximum likelihood estimate of the parameter was used in this plot. Superimposed on each plot is a least-squares, best-fit straight line. These plots indicate that modeling the gust-induced load with a normal distribution is inappropriate, and a gamma distribution models these loads fairly well.

Determining the 0.997 Load Enclosure

If the statistical distribution of the gust loads and the parameters of that distribution were known, it is straightforward to find a 0.997 load enclosure by integration. For normal data a standard normal probability table can be used, with the result that 99.7% of the data lies within three standard deviations of the mean. For a general

Presented as Paper 99-1253 at the AIAA/ASME/ASCE/AHS/ASC 40th Structures, Structural Dynamics, and Materials Conference, St. Louis, MO, 12–15 April 1999; received 15 December 1999; revision received 14 April 2000; accepted for publication 18 April 2000.

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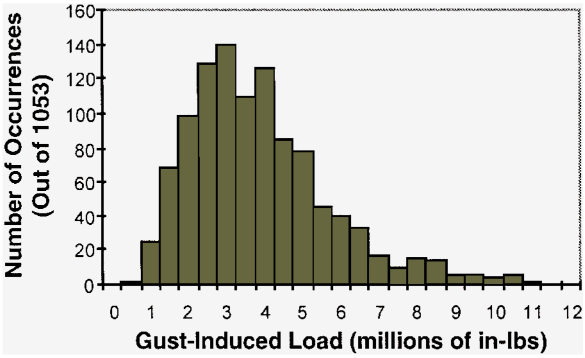

Fig. 1 Typical distribution of gust-induced load for a heavy-lift launch vehicle.

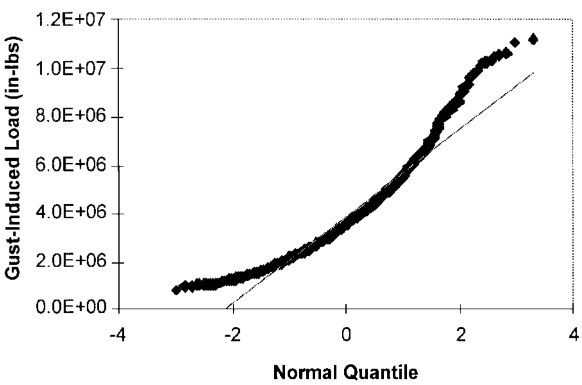

Fig. 2 Heavy-lift launch-vehicle gust-induced loads[3] normal probability plot.

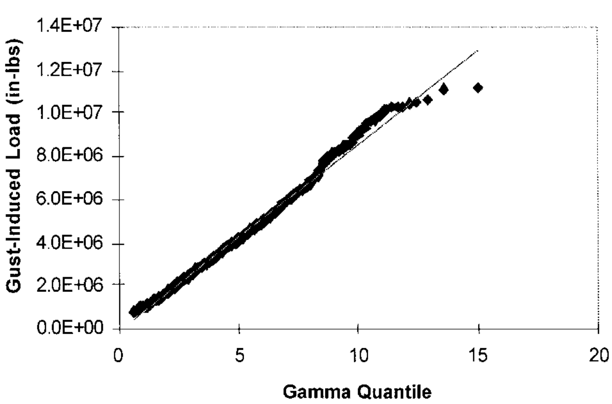

Fig. 3 Gamma probability plot for the data shown in Fig. 2.

probability density f_X the 0.997 load enclosure is the value of E, satisfying

$$\int_{-\infty}^{E} f_X(x)\,\mathrm{d}x = 0.997$$

If the true distribution is unknown (although the form of the distribution may be known, the parameters may not be), uncertainty in the load enclosure is introduced, and the 0.997 load enclosure can only be estimated. It is possible to place an upper confidence bound on this estimate, resulting in a one-sided tolerance bound on the load enclosure. In this work the goal is to find a 0.997/0.90 tolerance bound, i.e., a 90% upper confidence bound on the 0.997 load enclosure.

For normal data a p/C (e.g., 0.997/0.90) tolerance bound is of the form $\bar{x} + ks$, where k is based on the noncentral t distribution:

$$k = \left(t_{n-1,\sqrt{n}z_p,1-C}\right) / \sqrt{n}$$

Table 1 Coverage probability of bootstrapped 0.997/0.90 gamma tolerance intervals

Gamma shape parameter	Sample size 10	30	100
1.5	0.87	0.90	0.92
3.0	0.86	0.90	0.92
5.0	0.85	0.90	0.93
10.0	0.89	0.90	0.90

where $\sqrt{(n)}z_p$ is the noncentrality parameter of the distribution and z_p is the standard normal p-quantile (e.g., $z_{0.975} = 1.96$). Values of k are tabulated in Ref. 5 or can be readily computed.

Tolerance bounds for data other than normal data can be more difficult to determine. For gamma data two procedures are described. The first method, using a bootstrap statistical procedure, is useful for sample sizes under 1000 or so. The second procedure, useful for sample sizes greater than about 1000, is based on a normal transformation of the data.

Bootstrap Procedure

The bootstrapping procedure is a resampling procedure that is useful for determining confidence bounds on estimates.[10,11] Both nonparametric and parametric bootstrap procedures exist; a parametric procedure is used here to determine tolerance bounds for data having a two-parameter gamma distribution. Reference 10 describes the BC_a parametric bootstrap that is used here. The bootstrap and the BC_a method, in particular, have nice asymptotic properties, but in order to demonstrate the effectiveness of this procedure for estimating percentiles of gamma data based on relatively small samples, Monte Carlo simulations were performed. For sample sizes of 10, 30, and 100, and gamma shape parameters of 1.5, 3, 5, and 10 (the results are independent of the scale parameter of the gamma distribution), the coverage probabilities of the BC_a tolerance bounds were estimated. Table 1 summarizes the results of the Monte Carlo simulation. The table entries are the proportion of 1000 bootstrap 0.997/0.90 tolerance intervals that covered the true 0.997 quantile of a gamma distribution with given shape parameter and given sample size. In each case approximately 90% of the bootstrapped tolerance intervals include the true 0.997 quantile of the distribution, indicating that the BC_a tolerance bound results in accurate tolerance bounds.

Drawbacks to the parametric bootstrap procedure for calculating tolerance bounds include the following: 1) It is computer-intensive, taking a few minutes to perform for sample sizes on the order of 1000; and 2) it is sensitive to the assumption of a gamma distribution. The second drawback cannot be avoided by using a nonparametric bootstrap; it is not possible to estimate the 0.997 enclosure of a small set of data without making assumptions about the distribution. The first drawback is less of an issue.

Applying this method to the data shown in Fig. 1, we obtain a 0.997/0.90 tolerance bound of 10,960,000 in.-lb.

Nonparametric Procedure

A nonparametric method of calculating a tolerance bound can be used when the sample size is sufficiently large. This method relies on a standard result of probability theory that for a continuous probability distribution a nondecreasing function exists that, when applied to the data, transforms the data to a normal distribution: If G is the cumulative distribution function of a continuous random variable X and F_Z is the standard normal cumulative distribution function, then $f(X)$ has a standard normal distribution, where $f(\cdot) = F_Z^{-1}[G(\cdot)]$. See Fig. 4 for a graphical explanation of this result, which follows from the probability integral transformation.[12] This result is useful for estimating tolerance bounds because if the transforming function f can be accurately estimated, tolerance bounds for nonnormal data can be calculated using normal tolerance bounds.

To see this, suppose f is known. Let X be a random variable from a continuous distribution G so that $f(X) = Z$ is a standard normal random variable. Let $\{x_1, x_2, \ldots, x_n\}$ be a sample from G, and let T_Z be the p/C tolerance bound based on $\{f(x_1), f(x_2), \ldots, f(x_n)\}$,

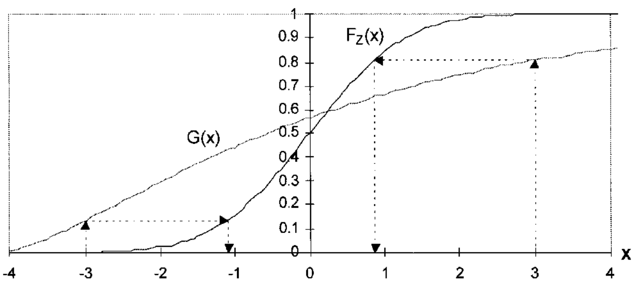

Fig. 4 Plot demonstrating the normalizing transformation. $F_Z(X)$ is the standard normal cumulative distribution function (Cdf), and $G(X)$ is the Cdf of a general distribution. The normalizing function in this case is the one that transforms, for example, -3 to -1.1 and 3 to 0.9.

using normal tolerance bound theory. Define z_p to be the p-quantile of a standard normal distribution. By the definition of a p/C tolerance bound, we have

$$C = P(T_Z > q_p)$$

$$= P\left[f^{-1}(T_Z) > f^{-1}(z_p)\right]$$

Let $T_X = f^{-1}(T_Z)$ and $q_p^X = f^{-1}(z_p)$, and we have

$$C = P\left(T_X > q_p^X\right)$$

If we can show that q_p^X is the p-quantile of the distribution of X, i.e.,

$$\int_{-\infty}^{q_p^X} g(x)\,\mathrm{d}x = p$$

then we have demonstrated that $T_X = f^{-1}(T_Z)$ is a p/C tolerance interval for X. This is easily shown as follows:

$$p = P(Z \le z_p)$$

$$= P\left[f^{-1}(Z) \le f^{-1}(z_p)\right]$$

$$= P\left(X \le q_p^X\right)$$

i.e., q_p^X is the p-quantile of X.

If there are sufficient data to estimate the normalizing function f, or its inverse f^{-1}, tolerance bounds for nonnormal data can be estimated by $f^{-1}(T_Z)$, where T_Z is the tolerance bound assuming a normal distribution (see Fig. 5). The k-value for a 0.997/0.90 normal tolerance interval is 2.83 for a sample of 1131. This corresponds to $1.05E+07$ for a 0.997/0.90 tolerance bound for the gust loads data shown in Fig. 5. This can be converted to a k-value by using the sample mean and standard deviation of the gust loads data. For a 0.997/0.90 tolerance bound estimation of the function f^{-1} in the vicinity of T_Z requires a sample of 1000 or more. For the gust loads, shown in Fig. 3, a fifth-order polynomial approximates f^{-1} well on a normal probability plot in the vicinity of T_Z.

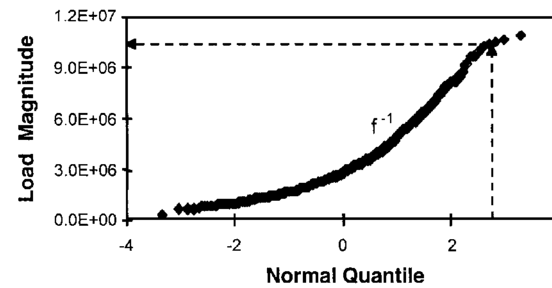

Fig. 5 Nonparametric procedure to establish tolerance bounds.

Applying this method to the data shown in Fig. 1, we obtain a 0.997/0.90 tolerance bound of 11,040,000 in.-lb.

Conclusions

We have presented two methods of finding tolerance bounds for gust loads analysis data. One assumes that the gusts follow a gamma distribution and are useful for smaller sample sizes. The other does not require this assumption, but requires considerably more data. For the data shown in Fig. 1, both methods result in similar tolerance bounds, approximately 11,000,000 in.-lb for each. The statistical analysis approach presented here was used to derive the 99.7/90 gust-induced loads in Ref. 3.

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